Models for Enhanced Absorption in Inhomogeneous Superconductors

Sergey V. Barabash, David Stroud
Department of Physics, The Ohio State University, Columbus, Ohio 43210
(February 1, 2008)

Abstract

We discuss the low-frequency absorption arising from quenched inhomogeneity in the superfluid density ρ_s of a model superconductor. Such inhomogeneities may arise in a high-T$_c$ superconductor from a wide variety of sources, including quenched random disorder and static charge density waves such as stripes. Using standard classical methods for treating randomly inhomogeneous media, we show that both mechanisms produce additional absorption at finite frequencies. For a two-fluid model with weak mean-square fluctuations $\langle(\delta\rho_s)^2\rangle$ in ρ_s and a frequency-independent quasiparticle conductivity, the extra absorption has oscillator strength proportional to the quantity $\langle(\delta\rho_s)^2\rangle/\rho_s$, as observed in some experiments. Similar behavior is found in a two-fluid model with anticorrelated fluctuations in the superfluid and normal fluid densities. The extra absorption typically occurs as a Lorentzian centered at zero frequency. We present simple model calculations for this extra absorption under conditions of both weak and strong fluctuations. The relation between our results and other model calculations is briefly discussed.

PACS numbers:

I. INTRODUCTION

Some high-T_c superconductors, such as $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$, absorb very strongly in the microwave regime even at temperatures T far below T_c [1,2]. At a fixed frequency as a function of T, the real conductivity typically has two features: a sharp peak near T_c, and a broad, frequency-dependent background extending to quite low T. The peak near T_c is usually ascribed to critical fluctuations arising from the superconducting transition.

The origin of the broad background is less clear. In conventional s-wave superconductors, there is no such background, because the material cannot absorb radiation at frequencies below the energy gap for pair excitations, 2Δ. But in high-T_c materials, which are thought to have a $d_{x^2-y^2}$ order parameter [3,4], the gap vanishes in certain nodal directions in k-space. Hence, gapless nodal quasiparticles can be excited at arbitrarily low T, and these can absorb at low frequencies. But the observed absorption appears to be stronger than expected from the quasiparticles alone in a two-fluid model [5].

Several authors have suggested quenched inhomogeneities as the origin of this extra absorption. Such inhomogeneities could arise from statistical fluctuations in the local densities of holes or impurities, or from the presence of superfluid-suppressing impurities such as Zn. Corson *et al* [1] found that such fluctuations in $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$displace about 30% of the spectral weight from the condensate to the conductivity at finite frequencies. In addition, several explicit models have been presented which produce such a displacement. For example, Van der Marel and Tsvetkov [6] have calculated the fluctuation conductivity due to a periodic one-dimensional modulation of the phase stiffness. The present authors [7] have calculated the fluctuation conductivity of a two-dimensional array of Josephson junctions with quenched randomness in the critical currents, using the Kubo formalism [8]. They found an excess contribution to the conductivity below T_c arising from this inhomogeneity, which arose from displacement of some of the superfluid contribution from zero to finite frequencies.

More recently, Orenstein [9] considered a two-fluid model of superconductivity with quenched, anticorrelated inhomogeneity in both the normal and superfluid densities; he found that some of the spectral weight of the conductivity is displaced from zero to finite frequencies, and gave explicit expressions for this extra conductivity. Han [10] has given a field-theoretic treatment of a randomly inhomogeneous two-dimensional d-wave superconductor, using a replica formalism to treat the assumed weak disorder. He obtained an extra Lorentzian peak in the real conductivity, centered at zero frequency, arising from this disorder, and a corresponding reduction in the superfluid density. All such models are made more plausible by imaging experiments which have directly observed spatial fluctuations in the local superconducting energy gap in underdoped cuprate superconductors [11].

In this paper, we study the low-frequency complex conductivity of several models for two-dimensional superconductors with quenched inhomogeneities. We use a straightforward approach to obtain the effective complex conductivity, based on standard treatments of randomly inhomogeneous media in the quasistatic regime [12]. From this approach, we obtain sum rules for the complex conductivity. We also find, in a number of cases, explicit expressions for the extra conductivity $\delta\sigma_e(\omega)$ due to the quenched inhomogeneities. Where the models are comparable, our explicit form for $\delta\sigma_e(\omega)$ reduces to that of Han, even though obtained using a quite different approach.

The remainder of this paper is organized as follows. In Section II, we develop sum rules for the optical conductivity of a two-dimensional superconductor with quenched inhomogeneities, and apply them to several explicit models. In Section III, we present some approximate calculations for the effective conductivity of such superconductors [13]. A concluding discussion is given in Section IV.

II. CONDUCTIVITY SUM RULES

A. Kramers-Kronig Relations

We consider a two-dimensional superconductor, in which the local conductivity has normal and superfluid contributions which act in parallel, but, in contrast to the usual two-fluid model, these terms are spatially varying. The complex local conductivity $\sigma(\mathbf{x}, \omega)$ is taken as a scalar function of position $\mathbf{x}$ and frequency ω of the following form:

$$\sigma(\omega, \mathbf{x}) = \sigma_n(\omega, \mathbf{x}) + \frac{i\rho_s(\mathbf{x})}{\omega}. \tag{1}$$

σ_n might be a contribution from the nodal quasiparticles expected in a superconductor with $d_{x^2-y^2}$ order parameter symmetry, while ρ_s represents the perfect-conductivity response of the superconductor. Our goal is to calculate the position-independent effective complex conductivity of this medium, which we denote

$$\sigma_e(\omega) \equiv \sigma_{e,1}(\omega) + i\sigma_{e,2}(\omega). \tag{2}$$

We first obtain a sum rule for $\sigma_{e,1}$. We start from the Kramers-Kronig relations obeyed by any function, such as $\sigma_e(\omega)$, which describes a causal response:

$$\sigma_{e,1}(\omega) = \frac{2}{\pi} P \int_0^\infty \frac{\omega' \sigma_{e,2}(\omega')}{\omega'^2 - \omega^2} d\omega' + \sigma_\infty \tag{3}$$

and

$$\sigma_{e,2} = \frac{\rho_{s,e}}{\omega} - \frac{2\omega}{\pi} P \int_0^\infty \frac{\sigma_{e,1}(\omega') - \sigma_\infty}{\omega'^2 - \omega^2} d\omega'. \tag{4}$$

Here P denotes the principal part, and $\sigma_\infty \equiv \mathrm{Lim}_{\omega\to\infty}\sigma_{e,1}(\omega)$. We have assumed that $\mathrm{Lim}_{\omega\to\infty}\sigma_n(\omega)$ position-independent, and have used the fact that, at high frequencies, $\sigma_{e,2} \to i\rho_{s,e}/\omega$, where $\rho_{s,e}$ is the effective superfluid density. At sufficiently large ω, the right hand side of eq. (4) takes the form

$$\sigma_{e,2} \to \frac{1}{\omega}\left(\rho_{s,e} + \frac{2}{\pi}\int_0^\infty [\sigma_{e,1}(\omega') - \sigma_\infty] d\omega'\right). \tag{5}$$

We first write down a perturbation result for $\sigma_e(\omega)$, which will be needed below. If the spatial fluctuations in $\sigma(\omega, \mathbf{x})$ are small in magnitude compared to it spatial average, σ_{av}, then to second order in the $|\delta\sigma|/|\sigma_{av}| \equiv |\sigma(\omega, \mathbf{x}) - \sigma_{av}(\omega)|/|\sigma_{av}(\omega)|$,

$$\sigma_e \approx \sigma_{av} - \frac{1}{2}\frac{\langle(\delta\sigma)^2\rangle}{\sigma_{av}}, \tag{6}$$

where $\langle ... \rangle$ denotes a space average of the quantity in brackets [12].

B. Frequency-Independent Normal Conductivity

Suppose first that $\sigma_n(\omega)$ is real, nonzero, frequency-independent, and spatially uniform. Then $\sigma_\infty = \sigma_n$, and, at high ω, however large the fluctuations in $\rho_s(\mathbf{x})$, the fluctuations in the *local* complex conductivity, $\sigma_n + i\rho_s(\mathbf{x})/\omega$, are small compared to the space-averaged conductivity $\sigma_{av}(\omega) = \sigma_n + i\rho_{s,av}/\omega$; we can thus use eq. (6). Since only ρ_s is fluctuating, and not σ_n, eq. (6) becomes

$$\sigma_e \approx \sigma_n + \frac{i\rho_{s,av}}{\omega} + \frac{1}{2}\left(\frac{\langle(\delta\rho_s)^2\rangle}{\omega^2\sigma_n + i\omega\rho_{s,av}}\right), \tag{7}$$

where $\delta\rho_s(\mathbf{x}) = \rho_s(\mathbf{x}) - \rho_{av}$. Thus, to leading order in $1/\omega$, $\sigma_{e,2} \sim \rho_{s,av}/\omega$. Equating this expression to the right-hand side of eq. (5), we obtain

$$\int_0^\infty \left[\sigma_{e,1}(\omega') - \sigma_n\right] d\omega' = \frac{\pi}{2}\left(\rho_{s,av} - \rho_{s,e}\right) \tag{8}$$

One consequence of eq. (8) is that, if $\rho_s(\mathbf{x})$ is spatially varying, $\rho_s(\mathbf{x})$, $\rho_{s,av} > \rho_{s,e}$, the right-hand side of eq. (8) is positive, and *there will be an additional contribution to $\sigma_{e,1}(\omega)$, above σ_n.*

The sum rule (8) requires only that the conductivity fluctuations are asymptotically small at large ω, and not necessarily that ρ_s have small fluctuations. If, however, the fluctuations in ρ_s are, in fact, small, then $\rho_{s,e}$ can be calculated approximately using the analog of eq. (6), namely,

$$\rho_{s,e} \approx \rho_{s,av} - \frac{1}{2}\frac{\langle(\delta\rho_s)^2\rangle}{\rho_{s,av}}. \tag{9}$$

From this approximate result, eq. (8) becomes

$$\int_0^\infty \left[\sigma_{e,1}(\omega') - \sigma_n\right] d\omega' \sim \frac{\pi}{4}\rho_{s,av}\left(\frac{\langle(\delta\rho_s)^2\rangle}{\rho_{s,av}^2}\right). \tag{10}$$

For fixed mean-square fluctuations $\langle(\delta\rho_s)^2\rangle/\rho_{s,av}^2$, this integral is proportional to the average superfluid density $\rho_{s,av}$. That is, the extra integrated fluctuation contribution to $\sigma_{e,1}$, above the mean contribution of the normal fluid, is proportional to $\rho_{s,av}$. A similar result has been reported in experiments carried out over a range of mean superfluid densities [1].

C. Frequency-Dependent, Spatially Fluctuating Normal Conductivity

Next, we consider a frequency-dependent $\sigma_n(\omega, \mathbf{x})$, assuming that $\mathrm{Lim}_{\omega\to\infty}\sigma_n(\omega, \mathbf{x}) = 0$, but allowing for a spatially varying $\sigma_n(\omega, \mathbf{x})$. Then eq. (4) becomes

$$\sigma_{e,2}(\omega) = \frac{\rho_{s,e}}{\omega} - \frac{2\omega}{\pi}P\int_0^\infty \frac{\sigma_{e,1}(\omega')}{\omega'^2 - \omega^2}d\omega' \to \frac{1}{\omega}\left(\rho_{s,e} + \frac{2}{\pi}\int_0^\infty \sigma_{e,1}(\omega')d\omega'\right), \tag{11}$$

where the last result is again valid at sufficiently large ω. We now assume a Drude form for $\sigma_n(\omega, \mathbf{x})$, i. e.,

$$\sigma_n(\omega, \mathbf{x}) = \frac{\rho_n(\mathbf{x})\tau_n(\mathbf{x})}{1 - i\omega\tau_n(\mathbf{x})}, \tag{12}$$

where $\rho_n(\mathbf{x})$ is a suitable normal fluid density, and $\tau_n(\mathbf{x})$ is a quasiparticle relaxation time. Then at sufficiently high ω,

$$\sigma_2(\omega, \mathbf{x}) \sim \frac{i[\rho_s(\mathbf{x}) + \rho_n(\mathbf{x})]}{\omega}, \tag{13}$$

while $\sigma_1(\omega, \mathbf{x})$ falls off at least as fast as $1/\omega^2$. Similarly, $\sigma_{e,1}$ also falls off at least as fast as $1/\omega^2$ at large ω. Hence, $\sigma_e \sim i\sigma_{e,2}$ at high frequencies, where

$$\sigma_{e,2}(\omega) = \frac{(\rho_s + \rho_n)_e}{\omega}. \tag{14}$$

Here $(\rho_s + \rho_n)_e$ denotes the effective superfluid density of a hypothetical material whose spatially varying local superfluid density is $\rho_s(\mathbf{x}) + \rho_n(\mathbf{x})$. Equating the coefficients of ω^{-1} on right hand sides of eqs. (11) and (14) yields

$$\int_0^\infty \sigma_{e,1}(\omega')d\omega' = \frac{\pi}{2}\left[(\rho_s + \rho_n)_e - \rho_{s,e}\right], \tag{15}$$

which gives the integrated spectral weight of $\sigma_{e,1}(\omega)$.

To calculate this weight, we first introduce $\sigma_{n,av} \equiv \langle\sigma_n\rangle$, and note that

$$\int_0^\infty \sigma_{n,1,av}(\omega')d\omega' = \frac{\pi\rho_{n,av}}{2}, \tag{16}$$

even if $\tau_n(\mathbf{x})$ is position-dependent. Hence,

$$\int_0^\infty \left[\sigma_{e,1}(\omega') - \sigma_{n,1,av}(\omega')\right] d\omega' = \frac{\pi}{2}\left[(\rho_s + \rho_n)_e - \rho_{s,e} - \rho_{n,av}\right]. \tag{17}$$

Thus $\sigma_{e,1}(\omega')$ again has some extra spectral weight beyond that of $\sigma_{n,av}(\omega)$.

We first estimate expression (17) in the small-fluctuation regime where $|\delta\rho_s| \ll \rho_{s,av}$, assuming that ρ_n is non-fluctuating. Following steps analogous to those leading up to eq. (10), we obtain

$$\int_0^\infty \left[\sigma_{e,1}(\omega') - \sigma_{n,1,av}(\omega')\right] d\omega' \to \frac{\pi}{4}\langle(\delta\rho_s)^2\rangle \left(\frac{1}{\rho_{s,av}} - \frac{1}{\rho_{s,av} + \rho_n}\right). \tag{18}$$

In the limit $\rho_n \gg \rho_{s,av}$, this expression reduces to the right-hand side of eq. (10). In this regime, the extra spectral weight is again proportional to $\rho_{s,av}$. A frequency-independent σ_n is a special case of this limiting behavior. In the opposite limit ($\rho_{s,av} \gg \rho_n$), the behavior is

$$\int_0^\infty \left[\sigma_{e,1,av}(\omega') - \sigma_{n,1}(\omega')\right] d\omega' \to \frac{\pi}{4}\rho_n \frac{\langle(\delta\rho_s)^2\rangle}{\rho_{s,av}^2}. \tag{19}$$

Thus, in this limit, for fixed mean-square fluctuations in ρ_s, the extra spectral weight is *not* proportional to $\rho_{s,av}$ [14].

If both ρ_s and ρ_n are spatially varying, eq. (17) can still be easily treated when spatial fluctuations in ρ_s and ρ_n are small. Then, to second order in these fluctuations,

$$(\rho_s + \rho_n)_e \approx \rho_{s,av} + \rho_{n,av} - \frac{1}{2}\frac{\langle[\delta(\rho_s + \rho_n)]^2\rangle}{\rho_{s,av} + \rho_{n,av}}, \tag{20}$$

while $\rho_{s,e}$ is given approximately by eq. (9). Substituting eqs. (20) and (9) into eq. (17) gives

$$\int_0^\infty [\sigma_{e,1}(\omega') - \sigma_{n,1,av}(\omega')]d\omega' = \frac{\pi}{4}\left[\frac{\langle(\delta\rho_s)^2\rangle}{\rho_{s,av}} - \frac{\langle[\delta(\rho_s + \rho_n)]^2\rangle}{\rho_{s,av} + \rho_{n,av}}\right]. \tag{21}$$

To interpret eq. (21), we assume that ρ_s and ρ_n are correlated according to $\rho_n = \lambda\rho_s$. Thus $\lambda = 1$ and $\lambda = -1$ represent perfect correlation and perfect anticorrelation. The predictions of eq. (21) are particularly striking in the latter case, as postulated in some models of inhomogeneity [9,10]. In this case the *sum* $\rho_s + \rho_n$ is *not* fluctuating. Then the second term on the right-hand side of eq. (21) vanishes, and the extra spectral weight due to the inhomogeneity is again proportional to $\langle(\delta\rho_s)^2\rangle/\rho_{s,av}$. In the opposite case $\lambda = 1$, the right-hand side of eq. (21) may be negative, i. e. the total spectral weight is *smaller* than that given by $\sigma_{n,1,av}$.

D. Tensor Conductivity

The preceding arguments apply equally well to a *tensor* conductivity, as expected if the superconducting layer contains charge stripes or other types of static charge density waves. In this case, the local conductivity should be a 2×2 tensor [15] of the form $\sigma^{\alpha\beta}(\omega, \mathbf{x}) = R^{-1}(\mathbf{x})\sigma^d(\omega)R(\mathbf{x})$, where $\sigma^d(\omega)$ is a diagonal 2×2 matrix with diagonal components $\sigma_A(\omega)$, $\sigma_B(\omega)$, and $R(\mathbf{x})$ is a 2×2 rotation matrix describing the local orientation of the stripes. If this phase has a domain structure, the stripes would have either of two orientations relative to the layer crystalline axes, with equal probability. In a macroscopically isotropic layer, this domain structure leads to an scalar effective conductivity $\sigma_e(\omega)$. If, for example, the conductivity along the jth principal axis is $\sigma_j(\omega) = \sigma_{n,j}(\omega) + i\rho_{s,j}/\omega$, then the superfluid inhomogeneity produces an extra spectral weight at finite ω given by eq. (17).

III. MODEL CALCULATIONS

A. Weak Inhomogeneity

We now supplement these sum rules with some explicit expressions for $\sigma_e(\omega)$, beginning with weak inhomogeneity, $|\delta\sigma| \ll \sigma_{av}$. In this case, σ_e is given by eq. (6). If only ρ_s is spatially fluctuating, this equation reduces to eq. (7) and

$$\sigma_{e,1}(\omega) \approx \sigma_n + \frac{1}{2}\langle(\delta\rho_s)^2\rangle\frac{\sigma_n}{\rho_{s,av}^2 + \omega^2\sigma_n^2}. \tag{22}$$

Thus, the extra absorption is a Lorentzian centered at $\omega = 0$, of half-width $\propto \rho_s/\sigma_n$, and strength $\propto \langle(\delta\rho_s)^2\rangle$. A result equivalent to eq. (22) has also been obtained by Han [10] using

a replica formalism. Indeed, even the parameters of the two results are identical, provided we interpret $\langle(\delta\rho_s)^2\rangle/2$ in our model as equal to the quantity $g\Lambda^2/\pi$ in the model of Ref. [10].

Next, we consider the case where both ρ_s and σ_n have weak spatial variations. We assume that σ_n is given by eq. (12), that ρ_n is spatially varying, but not τ_n, and that ρ_n and ρ_s are correlated according to the rule $\delta\rho_n = \lambda\delta\rho_s$. Then σ_e is given approximately by eq. (6), with

$$\sigma_{av} = \frac{\rho_{n,av}\tau_n}{1 - i\omega\tau_n} + \frac{i\rho_{s,av}}{\omega} \tag{23}$$

and

$$\langle(\delta\sigma)^2\rangle = \langle(\delta\rho_s)^2\rangle\left(\frac{i}{\omega} + \frac{\lambda\tau_n}{1 - i\omega\tau_n}\right)^2. \tag{24}$$

Some representative plots of $\sigma_{e,1}(\omega)$ resulting from eqs. (6), (23) and (24) are shown in Fig. 1 for $\lambda = 1$ (perfect correlation) $\lambda = -1$ (perfect anticorrelation), and $\lambda = 0$ (no fluctuations in $\delta\rho_n$). The case $\lambda = -1$ appears most plausible physically, since it implies that the sum $\rho_n + \rho_s$ is non-fluctuating. It also produces the largest increase in $\sigma_{e,1}$ at any given ω.

For $\omega\tau_n \ll 1$, eqs. (6), (23), and (24) lead to the analytical result

$$\sigma_{e,1}(\omega) \approx \sigma_{n,av} + \frac{\langle(\delta\rho_s)^2\rangle}{2}\frac{\sigma_{n,av} - 2\lambda\tau_n\rho_{s,av}}{\rho_{s,av}^2 + \omega^2\sigma_{n,av}^2}. \tag{25}$$

Here $\sigma_{n,av} \equiv \rho_{n,av}\tau_n$ and we have neglected corrections of order $(\omega\tau_n)^2$ in the numerator. Eq. (25) is again identical with Han's result [10], obtained using a replica formalism, if we make the identification $\langle(\delta\rho_s)^2\rangle/2 \leftrightarrow g\Lambda^2/\pi$.

B. Temperature-Dependent Superfluid Density; Effective-Medium Approximation

Next, we calculate $\sigma_e(\omega)$ using a simple model for the conductivity of a CuO_2 layer proposed in Ref. *et al* [5], but generalized to include inhomogeneities. We assume that $\sigma(\omega, \mathbf{x})$ is given by eq. (1), but for simplicity that it can have only two values, σ_A and σ_B, with probabilities p and $1-p$. The corresponding conductivities are $\sigma_j(\omega)$ (j = A, B), where

$$\sigma_j(\omega) = \sigma_n(\omega) + i\frac{\rho_{s,j}}{\omega} \tag{26}$$

Following Ref. [5], we take $\sigma_n(\omega) = \rho_n\tau_n/(1 - i\omega\tau_n)$. We assume $\rho_n = \alpha T$, for $T < T_c$, as expected for nodal quasiparticles in a $d_{x^2-y^2}$ superconductor, and $\rho_n(T > T_c) = \alpha T_c$. We also take $\tau_n^{-1} = \beta T$. This linear T-dependence may be expected if $1/\tau_n$ is determined primarily by quasiparticle-quasiparticle scattering in two dimensions [16], rather than impurity scattering, which might give a T-independent $1/\tau_n$. For the superfluid density, we write $\rho_{s,A} = \gamma\rho_{s,0}$, $\rho_{s,B} = \gamma^{-1}\rho_{s,0}$, where $\rho_{s,0}(T) = \rho_{s,0}(0)\sqrt{1 - 2\alpha T/\rho_{s,0}(0)}$. This form ensures that $\rho_{s,0}$ decreases linearly with T at low T, as observed experimentally [17], and that it vanishes at a critical temperature T_c as $\sqrt{T_c - T}$. We use the experimental values of the

parameters T_c, $\sigma_0 \equiv \rho_n \tau_n$, $\rho_{s,0}(0)$, α and β (as quoted in Ref. [5]) for our numerical estimates. Finally, we have arbitrarily assumed that the inhomogeneity parameter $\gamma = 3$.

We compute $\sigma_e(\omega)$ using the Bruggeman effective-medium approximation (EMA) [18]:

$$p\frac{\sigma_A(\omega) - \sigma_e(\omega)}{\sigma_A(\omega) + \sigma_e(\omega)} + (1-p)\frac{\sigma_B(\omega) - \sigma_e(\omega)}{\sigma_B(\omega) + \sigma_e(\omega)} = 0. \tag{27}$$

The physically relevant solution is obtained by requiring that $\sigma_e(\omega)$ be continuous in ω and that $\sigma_{e,1}(\omega) > 0$. We also arbitrarily assume that $p = 1/2$. Our form for $\rho_{s,A}$ and $\rho_{s,B}$ guarantees that the EMA solution $\rho_{s,e}(p = 1/2) = \sqrt{\rho_{s,A}\rho_{s,B}} = \rho_{s,0}$.

Fig. 2 shows the resulting $\sigma_{e,1}(\omega, T)$ for several frequencies ranging from 0.2 to 0.8 THz, the range measured in Ref. [5]. Also shown are $\int_{\omega_{min}}^{\omega_{max}} \sigma_{e,1}(\omega, T) d\omega$ for $\omega_{min}/(2\pi) = 0.2$THz and $\omega_{max}/(2\pi) = 0.8$THz. Finally, we plot $\sigma_{n,1}(\omega, T)$ for these frequencies, as well as $\int_{\omega_{min}}^{\omega_{max}} \sigma_{n,1} d\omega$. Clearly, $\sigma_{e,1}(\omega)$ is substantially increased beyond the quasiparticle contribution, because of spatial fluctuations in ρ_s.

C. Percolation Effects

We next consider $\sigma_{1,e}(\omega)$ near a percolative superconductor to normal (S-N) transition. Such a transition might occur, for example, in a single CuO_2 layer doped by a non-superconducting element such as Zn, as has already been discussed by several authors [19]. We assume $\sigma(\mathbf{x})$ to be given by eq. (1), with a position-independent Drude σ_n given by eq. (12), and a $\rho_s(\mathbf{x})$ which has one of two values, ρ_s or zero, with probabilities p and $1-p$. We calculate $\sigma_e(\omega)$ using the EMA, eq. (27). The EMA predicts an S-N transition at $p = 0.5 = p_c$. For all p on either side of p_c, $\sigma_{e,1}(p, \omega)$ has a peak at $\omega = 0$, whose height diverges and whose half-width decreases to zero, as $p \to p_c$. This behavior is illustrated in Fig. 3, where we plot $\sigma_{e,1}(p, \omega)$ for several $p > p_c$, as calculated in the EMA. The other parameters are indicated in the Figure caption. The behavior of $\sigma_{e,1}$ is qualitatively similar to that shown in Fig. 2, but there is additional critical behavior near the S-N transition, which is absent from the weakly disordered system.

More exact results near p_c can be obtained with the help of a standard scaling hypothesis [12,20]. For the present model, it takes the form

$$\frac{\sigma_e}{\sigma_>} = |\Delta p|^t F_\pm\left(\frac{\sigma_</\sigma_>}{|\Delta p|^{s+t}}\right). \tag{28}$$

Here $\sigma_<$ and $\sigma_>$ denote the complex conductivities in the S and N regions of the layer; $\Delta p = p - p_c$, s and t are standard percolation critical exponents, which depend on dimensionality, and possibly on other structural details of the layer; and $F_\pm(x)$ are characteristic scaling functions which apply above and below p_c. The expected behavior [12] is $F_+(x) \sim 1$ for $x \ll 1$, and $F_-(x) \sim x$ for $x \ll 1$, while $F_\pm(x) \sim x^{t/(s+t)}$ for $x \gg 1$. From these forms of the scaling functions, we can infer [20] that $\sigma_{e,1}$ has a peak at $\omega = 0$ whose half-width on both sides of the percolation threshold is

$$|\Delta\omega| \approx \frac{\rho_s}{\sigma_n}|\Delta p|^{s+t}, \tag{29}$$

which vanishes at the percolation threshold, where $|\Delta p| \to 0$. The height of this zero-frequency peak diverges according to the law $\sigma_{e,1}(p, \omega = 0) \to |\Delta p|^{-s}$ on either side of the percolation threshold. For two-dimensional bond percolation, $s = t \approx 1.30$, while in the EMA in any dimension, $s = t = 1$. Thus, we expect that $\sigma_{e,1}(p, \omega)$ is characterized near p_c by a line of diverging height, and half-width which goes to zero near p_c. Precisely this type of behavior is seen in the EMA curves of Fig. 3.

D. Tensor Conductivity

The results shown in Fig. 3 correspond to an in-plane scalar superfluid density $\rho_s(\mathbf{x})$ with a random spatial variation. If instead $\rho_s(\mathbf{x})$ were a 2×2 *tensor* with principal axes varying randomly with position (as might occur for random quenched domains of stripes), a similarly enhanced $\sigma_{e,1}$ would be expected. If the two principal components of the conductivity tensor are given by eq. (26), and the stripes can point with equal probability along either crystallographic direction, then $\sigma_e(\omega)$ is given in the EMA by eq. (27) with $p = 0.5$. The resulting $\sigma_{e,1}(\omega)$ would behave exactly like that shown in Fig. 2.

IV. DISCUSSION

It is useful to compare our results to those of previous workers, considering first the work of Han [10]. For weak fluctuations in ρ_s, and a frequency-independent non-fluctuating σ_n, our results are identical to his, provided we identify our parameter $\langle(\delta\rho_s)^2\rangle/2$ with his quantity $g\Lambda^2/\pi$. However, our formal derivation is more elementary than that of Ref. [10]. Because of this simplicity, we can easily consider a wide range of quenched inhomogeneities. Since the two approaches yield identical results for comparable types of inhomogeneities, we infer that our calculations (which are based on standard methods of treating random inhomogeneities in classical field equations such as those of electrostatics) can also be done using field-theoretic techniques. This field-theoretic approach may therefore be useful in treating other classical problems in heterogeneous media.

We briefly discuss relation between the present model and that of Ref. [7]. In that earlier paper, the CuO_2 layer was treated as an array of small "grains" coupled together by overdamped resistively shunted Josephson junctions with Langevin thermal noise; $\sigma_e(\omega, T)$ was then computed using the classical Kubo formalism. It consists of two parts: a sharp peak near the Kosterlitz-Thouless transition at T_c, due to breaking of vortex-antivortex pairs, and a broader contribution, for $T < T_c$, which exists only if the critical currents have quenched randomness. This second contribution corresponds to that considered here for spatially varying ρ_s. For T well below T_c, there are few vortices in the Josephson array; hence, the Josephson links in the model of Ref. [7] behave like inductors, and the array acts like an inhomogeneous LR network. This network is simply a discretized version of the two-fluid model considered here: the resistances R represent the normal quasiparticle channel, and the inductances L corresponds to the superfluid.

To summarize, we have shown that, in a superconducting layer with a spatially varying ρ_s, there is extra absorption beyond that from low-lying quasiparticles alone. Such quasiparticles are expected in a $d_{x^2-y^2}$ superconductor. For several models of weak inhomogeneity,

we have shown that the extra spectral weight at finite frequencies is proportional to the quantity $\langle(\delta\rho_s)^2\rangle/\rho_s$, as reported for $T < T_c$ in underdoped samples of $BiSr_2Ca_2Cu_2O_{8+x}$ [5]. Furthermore, in this regime, the extra weight generally appears as a Lorentzian centered at zero frequency, with half-width proportional to ρ_s/σ_n. Similar behavior is predicted for tensor inhomogeneities, as is expected in stripe geometries. We also predict that, near an inhomogeneous superconductor-normal transition, the height of the inhomogeneity peak in $\sigma_1(\omega)$ diverges while its half-width goes to zero. Thus, superfluid inhomogeneity may be the principal source of the extra absorption beyond the two-fluid model which is reported in the high-T_c cuprates.

V. ACKNOWLEDGMENTS

This work has been supported by NSF Grant DMR01-04987, and by the U. S./Israel Binational Science Foundation. We are most grateful for useful conversations with Prof. David Bergman.

REFERENCES

[1] J. Corson, R. Mallozzi, J. Orenstein, J. N. Eckstein, and I Bozovic, Nature (London) **398**, 221 (1999).

[2] S.-F. Lee, D. C. Morgan, R. J. Ormeno, D. M. Brown, R. A. Doyle, J. R. Waldram, and K. Kadowaki, Phys. Rev. Lett. **77**, 735 (1996).

[3] D. A. Wollman, D. J. Van Harlingen, W. C. Lee, D. M. Ginsberg, and A. J. Leggett, Phys. Rev. Lett. **71**, 2134 (1993).

[4] J. R. Kirtley, C. C. Tsuei, J. Z. Sun, C. C. Chi, L. S. Yu-Jahnes, A. Gupta, M. Rupp, and M. B. Ketchen, Nature **373**, 225 (1995).

[5] . J. Corson, J. Orenstein, S. Oh, J. O'Donnell, and J. N. Eckstein, Phys. Rev. Lett. **85**, 2569 (2000).

[6] D. van der Marel and A. Tsvetkov, Czech. J. Phys. **46**, 3165 (1996).

[7] S. Barabash, D. Stroud, and I.-J. Hwang, Phys. Rev. **B61**, R14924 (2000).

[8] See, for example, G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981), p. 194.

[9] J. Orenstein, cond-mat/0201049.

[10] J. H. Han, Phys. Rev. **B66**, 054517 (2002).

[11] D. M. Lang, V. Madhavan, J. E. Hoffman, E. W. Hudson, H. Eisaki, S. Uchida, and J. C. Davis, Nature **415**, 412 (2002).

[12] See, e. g., D. J. Bergman and D. Stroud, Solid State Physics **46**, 147 (1992). Eq. (3.1) of that paper gives an expression equivalent to (6) for the dielectric constant in three dimensions.

[13] A brief version of some of these calculations has been given elsewhere [S. V. Barabash and D. Stroud, cond-mat/0207646 (presented at ETOPIM-6, Snowbird, Utah, July 2002)].

[14] Eq. (17) is actually valid for any σ_n satisfying at large frequencies $\sigma_n(\omega) \sim i\rho_n/\omega$.

[15] E. W. Carlson, D. Orgad, S. A. Kivelson, and V. J. Emery, Phys. Rev. **B62**, 3422 (2000).

[16] For a discussion, see, e. g., Ya. M. Blanter, Phys. Rev. **B54**, 12807 (1996) and references cited therein.

[17] W. N. Hardy, D. A. Bonn, D. C. Morgan, R. X. Liang, and K. Zhang, Phys. Rev. Lett. **70**, 3999 (1993).

[18] D. A. G. Bruggeman, Ann. Phys. Leipzig **24**, 636 (1935); R. Landauer, J. Appl. Phys. **23**, 779 (1952).

[19] J. D. Chai, S. V. Barabash, and D. Stroud, Physica **C366**, 13 (2001) and references cited therein.

[20] D. Stroud and D. J. Bergman, Phys. Rev. **B25**, 2061 (1982).

FIGURES

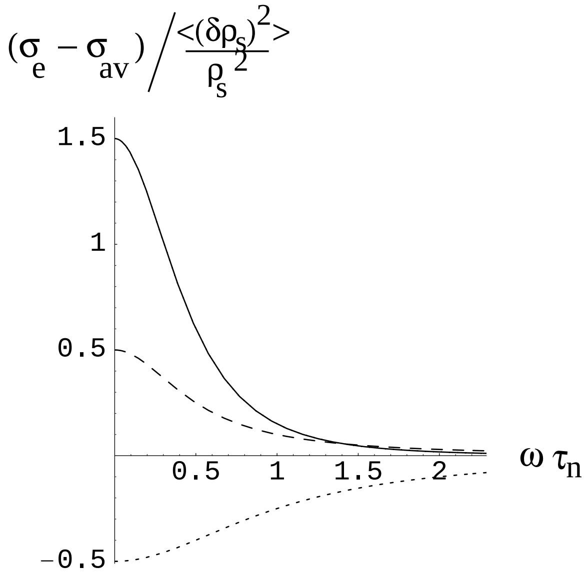

FIG. 1. Excess real part of effective conductivity, $\sigma_{e,1}(\omega) - \sigma_{n,1,av}(\omega)$, when there are weak correlated spatial fluctuations in both ρ_s and ρ_n, normalized by the relative magnitude of such fluctuations. $\sigma_{n,1,av} = \rho_{n,av}\tau_n/[1+\omega^2\tau_n^2]$ is the normal-state contribution. We assume $\delta\rho_n = \lambda\delta\rho_s$, and show results for $\lambda = -1$ (negative correlation, solid line), $\lambda = 1$ (positive correlation, dotted line), and $\lambda = 0$ (no fluctuations in the normal fluid density, dashed line). The average superfluid and normal fluid densities, $\rho_{s,av}$ and $\rho_{n,av}$, are assumed equal; the conductivities are in units of the quasiparticle $\rho_n\tau_n$.

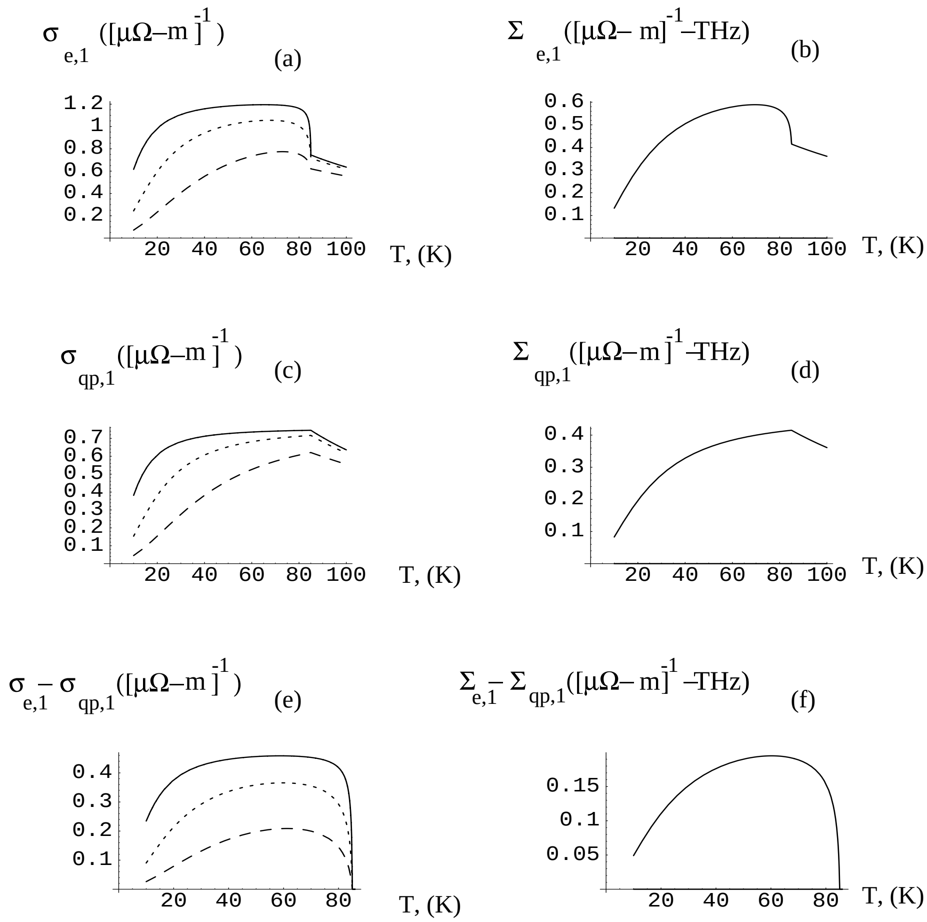

FIG. 2. (a) $\sigma_{e,1}(\omega, T)$, for $\omega/(2\pi) = 0.2$ THz (solid line), 0.4 THz (dotted line), and 0.8 THz (dashed line), for the model inhomogeneous superconductor described in the text. Also plotted are (b) $\Sigma_{e,1} \equiv \int_{\omega_{min}}^{\omega_{max}} \sigma_{e,1}(\omega) d\omega$, (c) $\sigma_{n,1}(\omega, T)$, (d) $\Sigma_{n,1} \equiv \int_{\omega_{min}}^{\omega_{max}} \sigma_{n,1}(\omega) d\omega$, (e) $\sigma_{e,1}(\omega, T) - \sigma_{n,1}(\omega, T)$, and (f) $\Sigma_{e,1} - \Sigma_{n,1} \equiv \int_{\omega_{min}}^{\omega_{max}} [\sigma_{e,1}(\omega) - \sigma_{n,1}(\omega)] d\omega$, where $\omega_{min}/(2\pi) = 0.2$Thz and $\omega_{max}/(2\pi) = 0.8$THz.

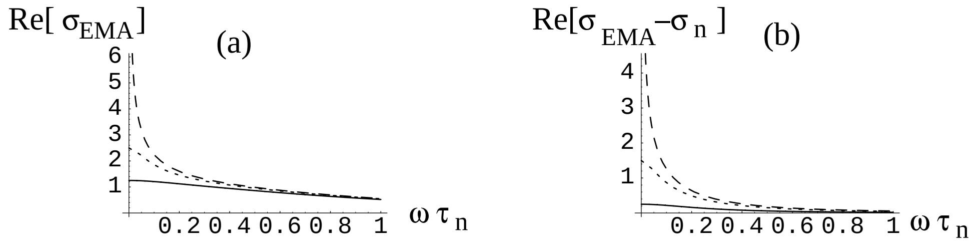

FIG. 3. Real part $\sigma_{e,1}(p, \omega)$, in units of the quasiparticle $\rho_n \tau_n$, of the effective conductivity of a two-dimensional composite of normal metal and superconductor, as calculated using the two-dimensional effective-medium approximation for several values of $p \geq p_c$: $p = 0.9$ (solid line), $p = 0.7$ (dotted line) and $p = 0.5$ (dashed line).